Precise Flattening of Cubic Bézier Segments

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2D Curve Rendering

- Flattening Methods
  - Forward differencing
  - Recursive subdivision
  - Parabolic Approximation (current method)

Comparison of Flattening Techniques

- Forward Differencing
  - Uniform interval in \( t \)
  - Too many or too few segments
- Recursive subdivision (RS)
  - Conservative achieved flatness
  - Requires achieved flatness evaluation
- Parabolic approximation (PA)
  - Minimal number of segments
  - Faster than RS

Approximating the start of a Bézier curve

Parabolic approximation

Inflection Point
Inflection Points

\[
\begin{align*}
    t_1 &= \frac{b_1 - a_1}{b_2 - a_2} \\
    t_2 &= \frac{d_1 - a_1}{d_2 - a_2} \\
    t_c &= \frac{a_1 b_2 - a_2 b_1}{a_1 d_2 - a_2 d_1}
\end{align*}
\]

Performance

- Performance measures
  - Relative number of generated segments
  - Relative execution time (C code)
- Test data (curve segment) set
  - Control points at (1,0), (0,0), and (0,1)
  - 4th control point on 100x100 grid (-3 to +3)
  - Flatness criterion at 0.0005

Ratio of generated segments (RS/PA)

PA Relative Achieved Flatness

RS Relative Achieved Flatness

Relative execution time (RS/PA)
Conclusion

- PA produces (on average) $2/3\dagger$ as many linear segments as $RS\ddagger$
- C-coded PA runs $37\%\ddagger$ faster than C-coded $RS$

$\dagger$ with flatness criterion being maintained within 4%
$\ddagger$ results are insensitive to chosen flatness criterion